Chapter 3 retake test

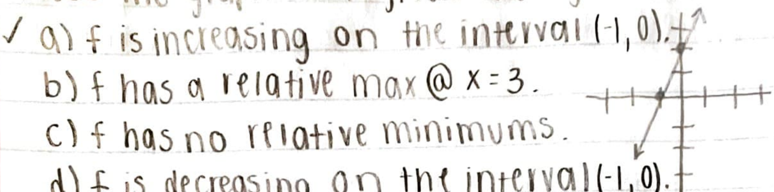
1) Use the graph of f' given the figure and choose the true statement.

✓ a) f is increasing on the interval (-1, 0).

b) f has a relative max @ x = 3.

c) f has no relative minimums.

d) f is decreasing on the interval (-1, 0).



$y = 2x + 2$

$y' = 2x$

$y' > 0$

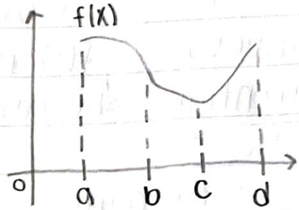
2) The graph f(x) is shown below. On which of the intervals is $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} > 0$?

a) $a < x < b$

b) $b < x < c$

✓ c) $c < x < d$

d) $b < x < d$



$\frac{dy}{dx} > 0 =$ increasing

$\frac{d^2y}{dx^2} > 0 =$ ∪ +

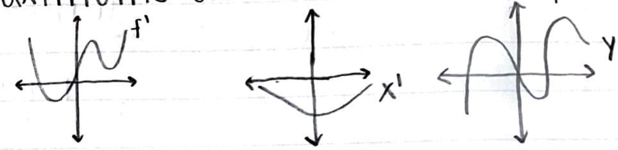
3) The graphs f', x', and y' are shown below. Which functions show relative maximums on the interval (b, c)?

a) f' and x'

✓ b) f' and y'

c) x' only

d) y' and x'



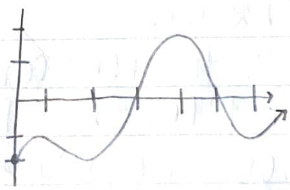
4) How many points of inflection does f'(x) have on the graph?

a) none

b) 3

c) 5

✓ d) 4



5) Find any relative maximums or minimums of $f(x) = 4x - x^2$.

a) Relative max @ (4, 2).

✓ b) Relative max @ (2, 4).

c) Relative min @ (-1, 2) and max @ (-2, 3).

d) None

$f'(x) = 4 - 2x$

$0 = 4 - 2x$

$0 = 2(2 - x)$

$0 \neq 2 \quad 0 = 2 - x$

$x = 2$

$f(2) = 4(2) - 2^2$

$f(2) = 4$

6) Find the value of c that is guaranteed by MVT (if it can by applied) for $f(x) = x^2 - 2x - 4$ on the interval $[-1, 2]$

✓ a) $c = 7/4$

b) $c = -7/2$

c) $c = 7/4, 7/2$

d) MVT does not apply.

1. Continuous b/c f(x) is a polynomial. ✓
2. Differentiable because f(x) is a polynomial ✓

MVT works!

① Slope of tangent

$f'(x) = 2x - 2$

② Slope of secant

$$\frac{f(1) - f(-1)}{1 - (-1)}$$

$$= \frac{[(-1)^2 - 2(-1) - 4] - [(2)^2 - 2(2) - 4]}{1 + 1}$$

$$= \frac{-1 - (-4)}{2} = \frac{3}{2}$$

③ $2c - 2 = 3/2$

$2c = 7/2$

$c = 7/4 \approx 1.75$

7) If Rolle's theorem can be applied, find all values for which $f'(c) = 0$. $f(x) = x^2 - 2x - 3$ on $[-1, 3]$

1. continuous b/c it's a polynomial. ✓
2. Differentiable b/c it's a polynomial. ✓
3. $f(-1) = (-1)^2 - 2(-1) - 3 = 0$ $f(-1) = f(3)$ ✓

$f(3) = (3)^2 - 2(3) - 3 = 0$ Rolle's can be applied

$f'(x) = 2x - 2$

$0 = 2x - 2$

$0 = 2(x - 2)$ $\boxed{c = 2}$

$0 \neq 2$ $0 = x - 2$

$x = 2$

8) Solve $\lim_{x \to 3} \frac{2(x-3)}{x^2-9}$ by using L'Hopital's rule

$\lim_{x \to 3} \frac{2(\cancel{x-3})}{(\cancel{x-3})(x+3)} = \lim_{x \to 3} \frac{2}{x+3} = \frac{2}{3+3} = \frac{2}{6} = \boxed{\frac{1}{3}}$

9) A farmer has to build a fence for his animals. Find the length and width of the rectangular fence that has an area of $121\ ft^2$ and a minimum perimeter.

Formula: $P = 2l + 2w$ constraint: $l \cdot w = 121$

$A = l \cdot w$ $w = 121/l$

$P = 2l + 2(121/l)$

$P = 2l + 242/l$

$P' = 2 - 242/l^2$

<u>Critical #'s:</u>

$0 = 2 - \frac{242}{l^2}$

$l^2(-2) = \left(-\frac{242}{\cancel{l^2}}\right)\cancel{l^2}$

$-2l^2 = -242$

$2l^2 = 242$

$x^2 = 121$

$x = \pm 11$

<u>Test</u>: $P'' = -242l^{-2}$ $P'' = \frac{484}{l^3}$

$P''(11) = 484/(11)^3 = 4/11 > 0$ $\cup$ (+)

$l = 11$

$w = 121/11$

$w = 11$ ✓

A maximum area of $121ft^2$ can be achieved when the dimensions of the fence are 11 ft by 11 ft.

A minimum perimeter is able to occur because $P'(11) = 0$ and $P''(11) = 0$.

10) Analyze and sketch $f(x) = x^3 - 3x^2 + 3$

$f'(x) = 3x^2 - 6x$

$f''(x) = 6x - 6$

X-int:

$0 = x^3 - 3x^2 + 3$

$-3 = x^3 - 3x^2$

Y-int:

$f(0) = 0^3 - 3(0)^2 + 3$

$f(0) = 3$ $(0,3)$

1st deriv test

$f'(x) = 3x^2 - 6x$

$0 = 3x^2 - 6x$

$0 = 3x(x-2)$

$x = 0, 2$

$f(0) = 0^3 - 3(0)^2 + 3 = 3$ $(0,3)$ max

$f(2) = 2^3 - 3(2)^2 + 3 = -1$ $(2,-1)$ min

2nd deriv test

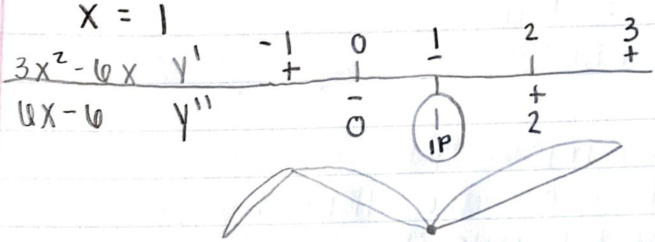
$f''(x) = 6x - 6$

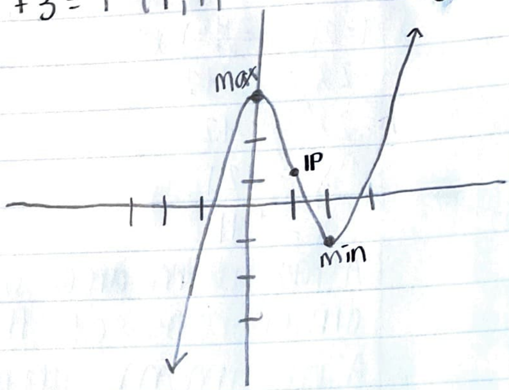
$0 = 6x - 6$

$0 = 6(x-1)$

$x = 1$

$f(1) = 1^3 - 3(1)^2 + 3 = 1$ $(1,1)$





There is a y-int @ $(0,3)$ because $f(0) = 3$. $f(x)$ is increasing on $(-\infty,-1) \cup (2,\infty)$ because $f'(-1) > 0$ and $f'(2) > 0$. $f(x)$ is decreasing on $(0,2)$ because $f'(0) < 0$. $f(x)$ is concave up on $(1,\infty)$ because $f'' > 0$ on the interval. $f(x)$ is concave down on $(-\infty,1)$ because $f'' < 0$ on that interval. There is a relative max @ $x = 0$ when $y = 3$ because $f'(0) = 0$. There is a relative min @ $x = 2$ when $y = -1$ because $f'(2) = 0$. There is an inflection point @ $(1,1)$ because $f''(1) = 0$ and f'' changes from negative to positive at this point

Answer Key

1) a
2) c
3) b
4) d
5) b
6) a